GROWTH KINETICS OF FIELD-INDUCED INVERSION DOMAINS IN SMECTIC-C FILMS

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Abstract

We numerically investigate the phase ordering kinetics of the smectic-C film under an external magnetic field. Due to the inversion symmetry of the director fields, external field induces Ising-like inversion domain walls in this system. It is observed that the usual dynamic scaling based on a single dominant length scale is violated for the Ising-like order parameter, except in the asymptotic time limit. This scaling violation is attributed to the existence of point defects along the domain walls, which provides a new length scale, the average separation between point defects, in addition to the usual length scale of average domain size.

Understanding the ordering kinetics of statistical systems subjected to a rapid thermal quench from a disordered phase to an ordered phase has long been one of the central issues in nonequilibrium statistical mechanics [1]. Recent non-perturvative approach [2] to this problem seems to provide a significant theoretical progress in this area. Being extended to systems possessing continuous degeneracy of ground states, the theory [3] has stimulated intensive theoretical and experimental investigations on the kinetics of systems having a variety of stable topological defects such as vortices, strings, and monopoles. Liquid crystal systems [4] have served as an appropriate test ground for important theoretical predictions made for phase ordering kinetics of systems possessing continuous symmetry [5].

One can imagine a non-equilibrium situation where a smectic-C film is quenched from an isotropic disordered phase under external magnetic fields. In this situation, the field-induced inversion domains will coarsen as the system evolves toward its equilibrium ordered phase. In the present work, we report a numerical investigation of growth kinetics of these inversion domain walls. A similar numerical investigation carried out by Pargellis et al. [6] was mostly concerned with the time decay of the point disclinations, leaving unexplored the scaling aspect in the coarsening of the inversion domains.

The elastic free energy for a smectic-C film under a magnetic field with the equal elastic constants is given by $F[n]=\int d^2r[K(\nabla\vec{n})^2/2+g(\vec{n}^2-1)^2/4-\chi(\vec{n}\cdot\vec{h})^2/2]$ where the director field $\vec{n}$ is a two-component real vector field, which characterizes the in-plane molecular orientation in a smectic-C film [4]. The presence of symmetry-breaking external field $\vec{h}$ breaks the $O(2)$ symmetry of the system and the inversion symmetry of the director field generates the discrete Ising-like double degeneracy of the ground states. Therefore, an appropriate order parameter can be defined as $\sigma=\vec{n}\cdot\hat{h}$ where $\hat{h}$ is the unit vector along the direction of the external magnetic field.

The evolution of the system toward equilibrium is assumed to be governed by the model A dynamics [7], appropriate to systems with non-conserved order parameter, of the form $\partial\vec{n}/\partial t=-\delta F/\delta\vec{n}$ where the thermal noise is ignored since we consider the case of zero-temperature quench only. Simulation is carried out by directly integrating this equation with random initial conditions using the Euler algorithm.

In order to see the scaling nature of the domain growth, we have tried to collapse the correlation functions $C(r,t)$ for the order parameter σ with an appropriate length scale $L_I(t)$ defined as $C(r=L_I(t),t)=1/2$ for a given time t. As Fig. 1 clearly shows, this procedure does not lead to a single curve onto which all correlation functions collapse. For comparison, we also show in Fig. 1 the scaling functions for pure $O(2)$ model and Ising model both in two dimensions obtained from recent simulations carried out by present authors [8]. We also see from Fig. 1 that the scaled correlation functions show considerable curvature near the origin at early stages of ordering and slowly approach the pure Ising correlation function, gradually reducing the short distance curvature as the ordering proceeds further. We may explain this slow approach to the asymptotic limit and the resulting scaling violation for any finite time span, in terms of the existence of a new length scale in the system in addtion to the usual length scale of average domain size. Namely, the point disclinations reside along the domain walls and the coarsening process involves not only the decay of the domain walls but also that of point disclinations on the walls, as clearly demonstrated in snapshots of Schilieren textures (Fig. 2). Hence the average separation between the point defects offers a new length scale to take into account in the ordering process, which can break the self-similarity of the ordering process based on a single length scale. The growth rate of average domain size is shown in Fig. 3. The average domain size $L_I(t)$ grows, at early stage, with the power law exponent of $\phi_1 \cong 0.38$ and then it slowly crosses over into the late time exponent $\phi_2 \cong 0.46$, which is close to the pure curvature-driven growth exponent of 1/2. The average separation between point disclinations $L_P(t)$ can be obtained from measuring the time dependence of the total number of point defects, which is shown in Fig. 4 for a few values of the external field. It exhibits a power law decay in time $N_P(t) \sim t^{-\nu_P}$ with $\nu_P \cong 0.76$, and this implies that the average separation between the point defects goes as $L_P(t) \sim N_P^{-1/2} \sim t^{0.38}$. Therefore, at the early stage of the ordering, the average domain size coincides with the average separation between the point defects and the short distance behavior of the correlation functions is expected to be strongly influenced by the presence of point defects on the walls, resulting in the scaled correlation functions with larger curvature near the origin. At the late stage of ordering, the average domain size grows faster than the average separation between the point defects does, and therefore the effect of the point defects on the short distance behavior of the correlation functions becomes weaker, reducing the short distance curvature of the scaled correlation functions.

Some of the results obtained in the present simulations may be tested experimentally in a free-standing smectic-C film under an external magnetic field.

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FIGURE CAPTIONS

Fig. 1: The short distance part of the scaled correlation functions which shows a slow approach to the pure Ising correlation function. Note that it has a considerable curvature at early times.

Fig. 2: Snapshots of the Schilieren texture at various times t = 10, 20, 40, 160 (from top to bottom) for h=0.4 on a square lattice with linear size 128. We see the annihilation of point disclinations on the domain walls as the ordering proceeds.

Fig. 3: Growth of the average domain size $L_I(t)$ for several external fields.

Fig. 4: Time dependence of the total number of point defects for a few values of the external magnetic field.

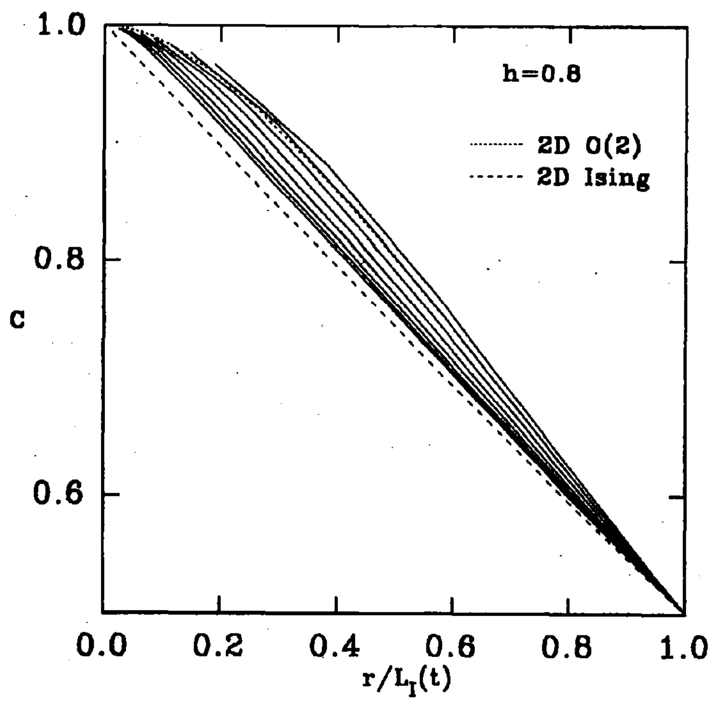

Fig. 1

Fig. 2

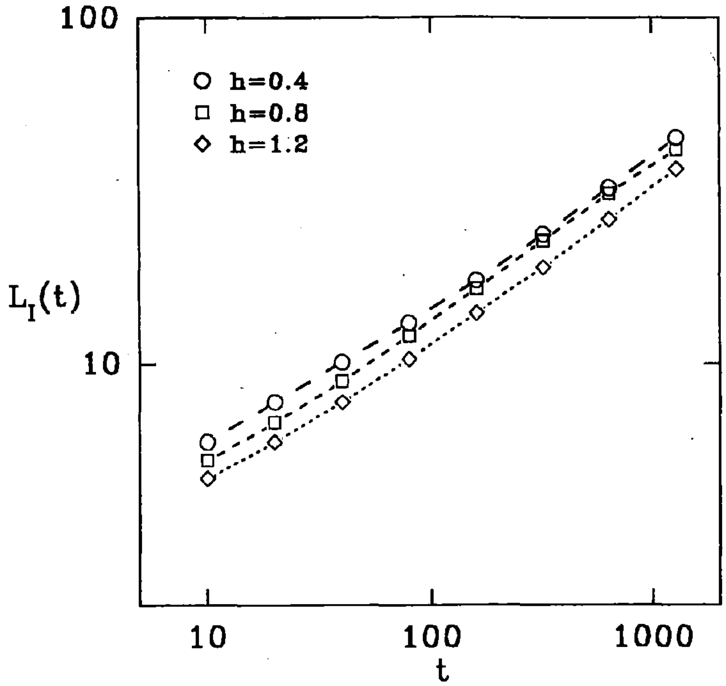

Fig. 3

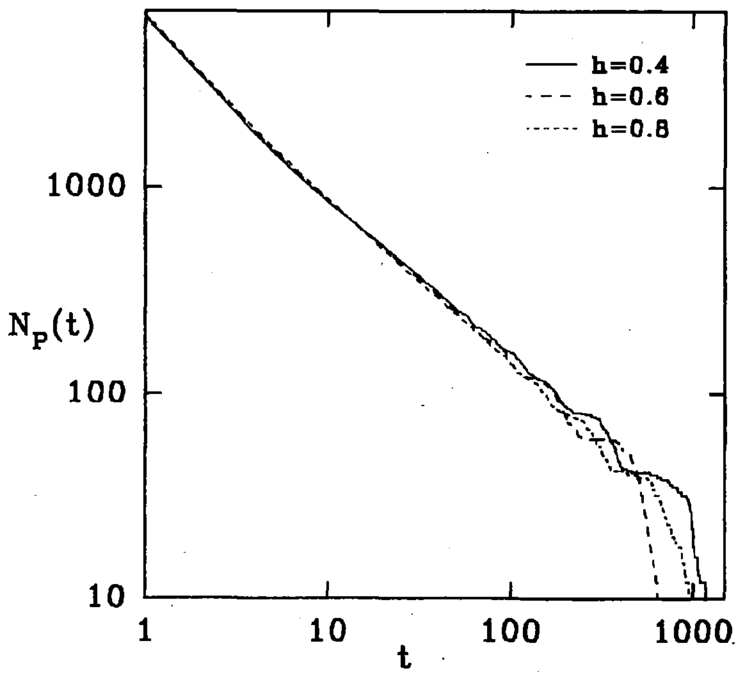

Fig. 4